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ON BOUNDED TRUTH–TABLE AND POSITIVE DEGREES*

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Ангел Дичев. ОБ ОГРАНИЧЕНО ТАБЛИЧНЫХ И ПОЗИТИВНЫХ СТЕПЕНЕЙ

В этом статье доказывается, что существует рекурсивно перечислимая btt-степень, которая содержит бесконечная антицепь из рекурсивно перечислимых p-степеней.

Angel Ditchev. ON BOUNDED TRUTH–TABLE AND POSITIVE DEGREES

In the present paper it is shown that there exists a recursively enumerable btt-degree containing an infinite anti-chain of recursively enumerable p-degrees.

In his dissertation [1] Degtev has studied the relationship between different tabular degrees. It is proved there that if $K^* = \{\mathrm{tt}, \mathrm{l}, \mathrm{p}, \mathrm{d}, \mathrm{c}, \mathrm{btt}, \mathrm{bl}, \mathrm{m}\}$, $\mathrm{r}, \mathrm{R} \in K^*$, $\mathrm{r} \neq \mathrm{R}$ and r is weaker than R, then every complete R-degree contains infinitely many recursively enumerable (r.e.) r-degrees. In connection with this he puts the questions, whether there exist a nonrecursive r.e. bp-degree, containing infinitely many bd-degrees, and a nonrecursive r.e. btt-degree, containing infinitely many r.e. bp-degrees. In [5] the first question is answered positively and in this paper the second question is answered positively too. Here, as in [5], something more is shown, namely, there exists an r.e. btt-degree containing an infinite anti-chain of r.e. p-degrees.

In connection with Degtev's questions, mentioned above, the following question arises: Let r and R be different tabular degrees such that R is not weaker than r.

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Does there exist an r.e. R-degree containing an infinite anti-chain of r-degrees. We expect that the answer of this question is positive.

In this paper we use $\mathbf{N}$ to denote the set of all natural numbers $\{0,1,2,\ldots\}$. Let $p_0 < p_1 < p_2\ldots$ be the sequence of all prime numbers and denote by M_i the set $\{x \mid \exists y(x = p_i.y)\}$, $i \in \mathbf{N}$.

If f is a partial function, then by $\mathrm{Dom}(f)$ we shall denote the domain of function f, and by $\mathrm{Ran}(f)$ the range of f. For any partial functions f and g by fg we shall denote the composition of f and g, i.e. $fg = \lambda x.f(g(x))$. If A is a finite set, we shall use $|A|$ to denote the cardinallity of the set A.

The sequence $A_0, A_1, \ldots$ ($\{A_k\}_{k\in\mathbf{N}}$) of sets of natural numbers is said to be recursive (r.e.) iff so is the set $\{(n,x) \mid x \in A_n\}$. The sequence $\varphi_0, \varphi_1, \ldots$ ($\{\varphi_k\}_{k\in\mathbf{N}}$) of unary functions is said to be total recursive (partial recursive) iff so is the binary function $\lambda i\lambda x.\varphi_i(x)$.

It would be useful to remind some definitions from [1, 5, 6].

If β is a Gödel function, then for natural numbers $k, p, p_1, \ldots, p_k, i$ we use the following notations:

$$\langle p_1,\ldots,p_k\rangle = \mu p[\beta(p,0) = k \,\&\, \beta(p,1) = p_1 \,\&\, \ldots \,\&\, \beta(p,k) = p_k];$$

$$\mathrm{lh}(p) = \beta(p,0); \quad (p)_i = \beta(p,i+1);$$

$$\mathrm{Seq}(p) \iff \forall x\{x < p \Rightarrow [\mathrm{lh}(x) \neq \mathrm{lh}(p) \vee \exists i(i < \mathrm{lh}(p) \,\&\, (x)_i \neq (p)_i)]\};$$

$$\mathrm{Seq}_k(p) \iff \mathrm{Seq}(p) \,\&\, \mathrm{lh}(p) = k.$$

The set A is called ***positively reducible*** (p-***reducible***) to the set B ($A \leq_{\mathrm{p}} B$) iff there exists a total recursive function f which satisfies the following conditions:

$$\text{(p)}\quad \begin{array}{l} \forall x\{\mathrm{Seq}(f(x)) \,\&\, \forall k[k < \mathrm{lh}(f(x)) \Rightarrow \mathrm{Seq}((f(x))_k)]\}; \\ \forall x\{x \in A \iff \exists k[k < \mathrm{lh}(f(x)) \,\&\, \forall i(i < \mathrm{lh}((f(x))_k) \Rightarrow ((f(x))_k)_i \in B)]\}. \end{array}$$

The set A is said to be ***truth table reducible*** (tt-***reducible***) to the set B iff there exists a total recursive function f which satisfies the following conditions:

$$\text{(tt)}\quad \begin{array}{l} \forall x\{x \in A \iff \exists k[k < \mathrm{lh}(f(x)) \,\&\, \forall i(i < \mathrm{lh}((f(x))_k) \Rightarrow \\ \qquad \{[(((f(x))_k)_i)_0 \in B \,\&\, \mathrm{Seq}(((f(x))_k)_i)] \vee \\ \qquad [((f(x))_k)_i \notin B \,\&\, \text{not } \mathrm{Seq}(((f(x))_k)_i)]\})]\}. \end{array}$$

If $\mathrm{r} \in \{\mathrm{p}, \mathrm{tt}\}$, then the set A is said to be br-***reducible*** to the set B ($A \leq_{\mathrm{br}} B$) iff there exists a natural number m and a total recursive function f which satisfy the conditions (r) and

$$\forall x[|\bigcup_k \bigcup_i ((f(x))_k)_i| \leq m].$$

If r is a reducibility, the set A is said to be r-***equivalent*** to the set B ($A \equiv_{\mathrm{r}} B$) iff $A \leq_{\mathrm{r}} B$ and $B \leq_{\mathrm{r}} A$.

For any reducibility r the family $d_{\mathrm{r}}(A) = \{B \mid B \equiv_{\mathrm{r}} A\}$ is called r-***degree*** of the set A.

The idea of constructing a btt-degree which contains infinitely many mutually incomparable p-degrees is the same as in [5] and comes from the proof of Skordev's conjecture [cf. 2, 3, 4]. Roughly speaking, we find btt-schemes which are not p-schemes. More precisely, we shall construct an r.e. sequence $\{B_k\}_{k\in\mathbb{N}}$ of sets of natural numbers such that the set B_i has the same btt-degree as the set B_j for any natural numbers i and j. At the same time, if $i \neq j$, then the sets B_i and B_j will be p-incomparable. For this aim we shall prove the following

Theorem 1. *There exist recursive sequences $\{\theta_{1,p}\}_{p\in\mathbb{N}}$, $\{\theta_{2,p}\}_{p\in\mathbb{N}}$ of total recursive functions such that for all natural numbers i and x the equivalences*

$$(*)\qquad \begin{aligned} x \in M_i &\iff \theta_{1,2i}(x) \in M_{i+1} \mathbin{\&} \theta_{2,2i}(x) \notin M_{i+1}, \\ x \in M_{i+1} &\iff \theta_{1,2i+1}(x) \in M_i \mathbin{\&} \theta_{2,2i+1}(x) \notin M_i \end{aligned}$$

hold, and such that if i and j are distinct and $(\varphi_1, \ldots, \varphi_{l_1}), \ldots, (\varphi_{l_{k-1}+1}, \ldots, \varphi_{l_k})$ is an arbitrary sequence of finite sequences of id *or compositions of $\theta_{1,0}, \theta_{2,0}, \theta_{1,1}, \theta_{2,1}, \ldots$, then there exists an $x \in \mathbb{N}$ such that the equivalence*

$$(**)\qquad \begin{aligned} x \in M_i \iff (&\varphi_1(x) \in M_j \mathbin{\&} \ldots \mathbin{\&} \varphi_{l_1}(x) \in M_j \vee \ldots \vee \\ &(\varphi_{l_{k-1}+1}(x) \in M_j \mathbin{\&} \ldots \mathbin{\&} \varphi_{l_k}(x) \in M_j) \end{aligned}$$

does not hold.

Proof. The construction of the sequences of functions $\{\theta_{1,p}\}_{p\in\mathbb{N}}$, $\{\theta_{2,p}\}_{p\in\mathbb{N}}$ we shall perform by steps. At step s we shall construct finite approximations $\theta^s_{k,p}$ of $\theta_{k,p}$, $k = 1, 2$; $p \in \mathbb{N}$, such that $\theta^s_{k,p} \subseteq \theta^{s+1}_{k,p}$ and $\theta^s_{k,p}(x)$ is a primitive recursive function (p.r.f.) of the variables s, p, x, $k = 1, 2$. At the end, we shall define $\theta_{k,p} = \bigcup_{s\in\mathbb{N}} \theta^s_{k,p}$, $k = 1, 2$; $p \in \mathbb{N}$.

We need some auxiliary definitions and lemmas.

Definition 1. Let $f_{1,0}, f_{2,0}, f_{1,1}, f_{2,1}, \ldots$ be an infinite sequence of unary functional symbols. Terms are defined by means of the following inductive clauses:

(a) Every symbol is a term;

(b) If τ_1 and τ_2 are terms, then $(\tau_1\tau_2)$ is a term.

We shall assume that there exist effective codings of all terms, of all finite sequences of terms, and of all finite sequences of finite sequences of terms.

Definition 2. We define the length $\mathrm{l}(\tau)$ of the term τ as follows:

(a) $\mathrm{l}(f_{k,i}) = 1$, $k = 1, 2$, $i \in \mathbb{N}$;

(b) If $\tau = (\tau_1\tau_2)$, then $\mathrm{l}(\tau) = \mathrm{l}(\tau_1) + \mathrm{l}(\tau_2)$.

Type and anti-type of a term are defined simultaneously by means of the following inductive definition:

Definition 3.

(a) If $\tau = f_{1,2i}$, then τ has type $i \to i + 1$;

(b) If $\tau = f_{1,2i+1}$, then τ has type $i + 1 \to i$;

(c) If $\tau = f_{2,2i}$, then τ has anti-type $i \to i + 1$;

(d) If $\tau = f_{2,2i+1}$, then τ has anti-type $i + 1 \to i$;

(e) If $\tau = (\tau_1\tau_2)$ and τ_2 has type $i \to k$ and τ_1 has type (anti-type) $k \to j$, then τ has type (anti-type) $i \to j$.

We say that τ has type (anti-type) iff τ has type (anti-type) $i \to j$ for some natural i, j; otherwise we say that τ has not type (anti-type).

Definition 4. Let $\mathfrak{A} = \langle \mathsf{N}; \theta_{1,0}, \theta_{2,0}, \theta_{1,1}, \theta_{2,1}, \dots \rangle$ be a partial structure. The value $\tau_{\mathfrak{A}}$ of the term τ in the structure $\mathfrak{A}$ we define as follows:

(a) If $\tau = f_{k,i}$, then $\tau_{\mathfrak{A}} = \theta_{k,i}$, $k = 1, 2$; $i \in \mathsf{N}$;

(b) If $\tau = (\tau^1\tau^2)$, then $\tau_{\mathfrak{A}} = \tau^1_{\mathfrak{A}}\tau^2_{\mathfrak{A}}$.

Lemma 1. *Let* $\mathfrak{A} = \langle \mathsf{N}; \theta_{1,0}, \theta_{2,0}, \theta_{1,1}, \theta_{2,1}, \dots \rangle$ *be a partial structure such that* $\theta_{1,i}$, $\theta_{2,i}$ *are finite functions and for all natural numbers* i *and* x *the following conditions hold:*

(a) $x \in M_i \cap \mathrm{Dom}(\theta_{1,2i}) \Rightarrow \theta_{1,2i}(x) \in M_{i+1}$;

(b) $x \in M_{i+1} \cap \mathrm{Dom}(\theta_{1,2i+1}) \Rightarrow \theta_{1,2i+1}(x) \in M_i$;

(c) $x \in M_i \cap \mathrm{Dom}(\theta_{2,2i}) \Rightarrow \theta_{2,2i}(x) \notin M_{i+1}$;

(d) $x \in M_{i+1} \cap \mathrm{Dom}(\theta_{2,2i+1}) \Rightarrow \theta_{2,2i+1}(x) \notin M_{i+1}$;

(e) $x \in [\mathrm{Dom}(\theta_{1,2i}) \cap \mathrm{Dom}(\theta_{2,2i})] \setminus M_i \Rightarrow \theta_{1,2i}(x) \notin M_{i+1} \vee \theta_{2,2i}(x) \in M_{i+1}$;

(f) $x \in [\mathrm{Dom}(\theta_{1,2i+1}) \cap \mathrm{Dom}(\theta_{2,2i+1})] \setminus M_{i+1} \Rightarrow$
$$\theta_{1,2i+1}(x) \notin M_i \vee \theta_{2,2i+1}(x) \in M_i.$$

Then for every term τ *which has type (anti-type)* $i_0 \to j_0$ *and for every partial structure* $\mathfrak{A} = \langle \mathsf{N}; \theta'_{1,0}, \theta'_{2,0}, \theta'_{1,1}, \theta'_{2,1}, \dots \rangle$ *such that*

$\theta'_{k,i}$ is an extension of $\theta_{k,i}$,

$\theta'_{k,i}$ satisfies the conditions (a)–(f) (when we replace $\theta'_{k,i}$ instead of $\theta_{k,i}$),

$k = 1, 2$; $i \in \mathsf{N}$ and

$x_0 \in M_{i_0} \cap \mathrm{Dom}(\tau_{\mathfrak{A}'})$

it holds $\tau_{\mathfrak{A}'}(x_0) \in M_{j_0}$ $\quad(\tau_{\mathfrak{A}'}(x_0) \notin M_{j_0})$.

Proof. By induction on the length $\mathrm{l}(\tau)$ of the term τ.

If $\theta_{1,0}$, $\theta_{2,0}$, $\theta_{1,1}$, $\theta_{2,1}$, ... are total functions, then the conditions (a)–(f) of Lemma 1 ensure the equivalences $(*)$ to be true for all natural numbers i and x.

Lemma 2. *For every partial structure*

$$\mathfrak{A} = \langle \mathsf{N}; \theta_{1,0}, \theta_{2,0}, \theta_{1,1}, \theta_{2,1}, \dots \rangle,$$

for all natural numbers $i_1, \dots, i_l$; $j_1, \dots, j_l$; $x_1, \dots, x_l$; $y_1, \dots, y_l$, *for every sequence of terms* $\tau^1, \dots, \tau^l$ *such that the following nine properties hold:*

1) $\theta_{1,i}$, $\theta_{2,i}$ *are finite functions,* $i \in \mathsf{N}$;

2) *For all natural numbers* i *and* x *the six conditions* (a)–(f) *of Lemma 1 hold;*

3) *If* τ^p *has type* $i \to j$, *then* $i \neq j$ *or* $j = j_p$;

4) *If* τ^p *has anti-type* $i \to j$, *then* $i \neq j$ *or* $j \neq j_p$;

5) *If* $1 \leqq p, q \leqq l$, $\tau\tau^p = \tau^q$, $x_p = x_q$, *and* τ^p *has type* $i \to j$, *then* $i \neq j_p$ *or* $j = j_q$;

6) *If* $1 \leqq p, q \leqq l$, $\tau\tau^p = \tau^q$, $x_p = x_q$, *and* τ^p *has an anti-type* $i \to j$, *then* $i \neq j_p$ *or* $j \neq j_q$;

7) $x_p \in M_{i_p} \setminus \left[\left(\bigcup_{i\in\mathsf{N}} \mathrm{Dom}(\theta_{1,i})\right) \cup \left(\bigcup_{i\in\mathsf{N}} \mathrm{Dom}(\theta_{2,i})\right)\right]$, $p = 1, \dots, l$;

8) $y_p \in M_{j_p} \setminus \left[\left(\bigcup_{i\in\mathsf{N}} \mathrm{Dom}(\theta_{1,i})\right) \cup \left(\bigcup_{i\in\mathsf{N}} \mathrm{Dom}(\theta_{2,i})\right)\right]$, $p = 1, \dots, l$;

9) *If* $x_p = x_q$ *and* $\tau^p = \tau^q$, *then* $y_p = y_q$,

there exists a partial structure $\mathfrak{A}' = \langle \mathsf{N}; \theta'_{1,0}, \theta'_{2,0}, \theta'_{1,1}, \theta'_{2,1}, \ldots \rangle$ *such that* $\theta'_{k,i}$ *is a finite extension of* $\theta_{k,i}$, $\theta'_{k,i}$ *satisfies the conditions* (a)–(f), $k = 1, 2$; $i \in \mathsf{N}$, $x_p \in \mathrm{Dom}(\tau^p_{\mathfrak{A}'})$ *and* $\tau^p_{\mathfrak{A}'}(x_p) = y_p$, $p = 1, \ldots, l$.

Proof. By induction on $\max\big(l(\tau^1), \ldots, l(\tau^l)\big)$.

First, let $\max\big(l(\tau^1), \ldots, l(\tau^l)\big) = 1$. Then τ^p has either a type or an anti-type. If for example $\tau^p = f_{1,2i}$, then $i_p \neq i$ or $j_p = i + 1$, and if for example $\tau^p = f_{2,2i+1}$, then $i + 1 \neq i_p$ and $j_p \neq i$, $p = 1, \ldots, l$. We can assume that if $p \neq q$, $1 \leqq p, q \leqq l$, then $\tau^p \neq \tau^q$.

If $f_{k,i} = \tau^p$, then we define $\theta'_{k,i}(x_p) = y_p$ and $\theta'_{k,i}(x) = \theta_{k,i}(x)$ for $x \neq x_p$, and if $f_{k,i} \notin \{\tau^1, \ldots, \tau^p\}$, then $\theta'_{k,i} = \theta_{k,i}$. It is easy to check that the structure $\mathfrak{A}' = \langle \mathsf{N}; \theta'_{1,0}, \theta'_{2,0}, \theta'_{1,1}, \theta'_{2,1}, \ldots \rangle$ is the needed.

Let us assume that Lemma 2 is true whenever $\max\big(l(\tau^1), \ldots, l(\tau^l)\big) \leqq n$, and $\tau^1, \ldots, \tau^l$ be terms such that $\max\big(l(\tau^1), \ldots, l(\tau^l)\big) = n+1$. We consider all those p such that $x_p = x_1$ and τ^p has the same last symbol as τ_1. For the sake of simplicity we can assume that $x_1 = x_2 = \cdots = x_l$ and $\tau^p = \tau''^p f_{1,2i}$ for some term τ''^p or $\tau^p = f_{1,2i}$, $p = 1, \ldots, l$. If for some p, $1 \leqq p \leqq l$, $\tau^p = f_{1,2i}$, then we define $\theta''_{1,2i}(x_p) = y_p$ and $\theta''_{1,2i}(x) = \theta_{1,2i}(x)$ for $x \neq x_p$; $\theta''_{k,j} = \theta_{k,j}$ for $(k, j) \neq (1, 2i)$.

If $\tau^p = \tau''^p f_{1,2i}$ for some term τ''^p, $p = 1, \ldots, l$, then let x'' be a natural number satisfying the conditions:

— $x'' \notin \{x_1, \ldots, x_l, y_1, \ldots, y_l\} \cup \Big(\bigcup_{i \in \mathsf{N}} \mathrm{Dom}(\theta_{1,2i})\Big) \cup \Big(\bigcup_{i \in \mathsf{N}} \mathrm{Dom}(\theta_{2,2i})\Big)$;

— if $x_1 \in M_i$, then $x'' \in M_{i+1}$;

— if $x_1 \notin M_i$, then $x'' \notin M_q$ for such a q that τ''^p has the last symbol which has the type or anti-type $q \to r$ for some r.

We define $\theta''_{1,2i}(x_1) = x''$, $x''_1 = \cdots = x''_l = x''$ and $\theta''_{1,2i}(x) = \theta_{1,2i}(x)$ for $x \neq x_1$; $\theta''_{k,j} = \theta_{k,j}$ for $(k, j) \neq (1, 2i)$. Then for the structure $\mathfrak{A}'' = \langle \mathsf{N}; \theta''_{1,0}, \theta''_{2,0}, \theta''_{1,1}, \theta''_{2,1}, \ldots \rangle$, for the natural numbers $i_1, \ldots, i_l$; $j_1, \ldots, j_l$; $x_1, \ldots, x_l$; $y_1, \ldots, y_l$, for the terms $\tau''_1, \ldots, \tau''_l$ the conditions of Lemma 2 are satisfied and $\max\big(l(\tau''^1), \ldots, i(\tau''^l)\big) \leqq n$. Then, according the assumption, there exists such a structure $\mathfrak{A}' = \langle \mathsf{N}; \theta'_{1,0}, \theta'_{2,0}, \theta'_{1,1}, \theta'_{2,1}, \ldots \rangle$ which we need, and Lemma 2 is proved.

Let us return to the proof of the Theorem 1.

At step s we shall define a partial structure $\mathfrak{A}^s = \langle \mathsf{N}; \theta^s_{1,0}, \theta^s_{2,0}, \theta^s_{1,1}, \theta^s_{2,1}, \ldots \rangle$, and at the end we shall define the structure $\mathfrak{A} = \langle \mathsf{N}; \theta_{1,0}, \theta_{2,0}, \theta_{1,1}, \theta_{2,1}, \ldots \rangle$.

By the even steps $s = 2n$ we shall ensure that the function $\theta_{k,i}$ is total, $k = 1, 2$; $i \in \mathsf{N}$. By the odd steps $s = 2n + 1$, if $n = \langle n_0, i, j \rangle$, $i \neq j$ and n_0 is a number of the finite sequence $(\tau^1, \ldots, \tau^{l_1}), \ldots, (\tau^{l_{k-1}+1}, \ldots, \tau^{l_k})$ of finite sequences of terms, and φ_p is the value of the term τ^p, $p = 1, \ldots, l_k$, we shall find an x such that the equivalence (**) does not hold, i. e. for some x we shall satisfy at least one of the following two conditions:

$$\text{(i) } x \in M_i \ \& \ \Big[\big(\varphi_1(x) \notin M_j \vee \ldots \vee \varphi_{l_1}(x) \notin M_j\big) \ \& \ldots \& \ \big(\varphi_{l_{k-1}+1}(x) \notin M_j \vee \ldots \vee \varphi_{l_k}(x) \notin M_j\big)\Big];$$

(ii) $x \notin M_i$ & $\Big[(\varphi_1(x) \in M_j$ & $\dots$ & $\varphi_{l_1}(x) \in M_j) \vee \dots \vee$

$(\varphi_{l_{k-1}+1}(x) \in M_j$ & $\dots$ & $\varphi_{l_k}(x) \in M_j)\Big]$.

Let us describe the construction.

Case I. $s = 2n$.

We define $\theta^s_{k,p}(x) = \theta^s_{k,p}(x)$ for $x \neq n$ and:

$$\text{if } n \notin \mathrm{Dom}(\theta_{1,2i}), \text{ then } \theta_{1,2i}(n) = \begin{cases} p_{i+1}, & \text{if } n \in M_i, \\ p_i, & \text{otherwise;} \end{cases}$$

$$\text{if } n \notin \mathrm{Dom}(\theta_{1,2i+1}), \text{ then } \theta_{1,2i+1}(n) = \begin{cases} p_i, & \text{if } n \in M_{i+1}, \\ p_{i+1}, & \text{otherwise;} \end{cases}$$

$$\text{if } n \notin \mathrm{Dom}(\theta_{2,2i}), \text{ then } \theta_{2,2i}(n) = \begin{cases} p_i, & \text{if } n \in M_i, \\ p_{i+1}, & \text{otherwise;} \end{cases}$$

$$\text{if } n \notin \mathrm{Dom}(\theta_{2,2i+1}), \text{ then } \theta_{2,2i+1}(n) = \begin{cases} p_{i+1}, & \text{if } n \in M_{i+1}, \\ p_i, & \text{otherwise.} \end{cases}$$

Case II. $s = 2n + 1$.

If not $\mathrm{Seq}(n)$ or $(n)_1 = (n)_2$, then we do nothing, i. e. $\theta^s_{k,p} = \theta^{s-1}_{k,p}$, $k = 1, 2$, $p \in \mathbb{N}$. If $n = \langle n_0, i, j\rangle$ and $i \neq j$, let n_0 be the number of the finite sequence of finite sequences of terms $(\tau^1, \dots, \tau^{l_1})$, $\dots$, $(\tau^{l_{k-1}+1}, \dots, \tau^{l_k})$. We consider two subcases:

Subcase A. There exists a natural number m, $0 \leqq m < k$, such that if $l_m + 1 \leqq p, q \leqq l_{m+1}$ and $\tau\tau^p = \tau^q$, then for some term τ has a type (anti-type) $k \to l$, then $k \neq j$ or $l = j$ ($k \neq j$ or $l \neq j$). We can assume that $m = 0$ and $l_m = 0$.

Let i_0 be an integer such that $i_0 \neq i$ and $i_0 \neq i'$, and τ^p has a type or anti-type $i' \to j'$ for some j', $1 \leqq p \leqq l_1$. Let in addition

$$i_1 = \dots = i_{l_1} = i_0, \quad j_1 = \dots = j_{l_1} = j,$$

$$x_1 = \dots = x_{l_1} = x_0 \in M_{i_0} \setminus \Big[\Big(\bigcup_{i\in\mathbb{N}} \mathrm{Dom}(\theta^{s-1}_{1,2i})\Big) \cup \Big(\bigcup_{i\in\mathbb{N}} \mathrm{Dom}(\theta^{s-1}_{2,2i})\Big)\Big],$$

$$y_1, \dots, y_l \in M_j \setminus \Big[\Big(\bigcup_{i\in\mathbb{N}} \mathrm{Dom}(\theta^{s-1}_{1,2i})\Big) \cup \Big(\bigcup_{i\in\mathbb{N}} \mathrm{Dom}(\theta^{s-1}_{2,2i})\Big)\Big].$$

It is easy to check that the structure $\mathfrak{A}^{s-1} = \langle \mathbb{N}; \theta^{s-1}_{1,0}, \theta^{s-1}_{2,0}, \theta^{s-1}_{1,1}, \theta^{s-1}_{2,1}, \dots\rangle$, the natural numbers $i_1, \dots, i_{l_1}$; $j_1, \dots, j_{l_1}$; $x_1, \dots, x_{l_1}$; $y_1, \dots, y_{l_1}$, the terms $\tau^1, \dots, \tau^{l_1}$ satisfy the conditions of Lemma 2. Therefore, there exists such a structure $\mathfrak{A}^s = \langle \mathbb{N}; \theta^s_{1,0}, \theta^s_{2,0}, \theta^s_{1,1}, \theta^s_{2,1}, \dots\rangle$ that $\theta^s_{k,p}$ satisfies the conditions (a)–(f) (if we replace $\theta^s_{k,p}$ instead of $\theta_{k,p}$), $k = 1, 2$, $p \in \mathbb{N}$ and $\tau^p_{\mathfrak{A}^s}(x_p) = y_p \in M_j$, $p = 1, \dots, l_1$. It is clear that this structure can be defined effectively.

Subcase B. Assume that subcase A does not hold.

Then we do nothing, i. e. $\theta^s_{k,p} = \theta^{s-1}_{k,p}$, $k = 1, 2$, $p \in \mathbb{N}$.

The construction is completed.

It is easy to check that the following lemmas are correct.

Lemma 3. *$\theta_{k,p}$ is a total recursive function, $k = 1, 2$, $p \in \mathbb{N}$.*

Lemma 4. *$\theta_{k,p}$ satisfies the equivalences $(*)$, $k = 1, 2$, $p \in \mathbb{N}$.*

Lemma 5. *If $i \neq j$ and $(\varphi_1, \ldots, \varphi_{l_1}), \ldots, (\varphi_{l_{k-1}+1}, \ldots, \varphi_{l_k})$ is an arbitrary sequence of finite sequences of* id *or compositions of the functions $\theta_{1,0}, \theta_{2,0}, \theta_{1,1}, \theta_{2,1}, \ldots$, then there exists an x such that (**) does not hold.*

The proof of Theorem 1 is completed.

Let $\varphi_{k,i}(x) = \langle i, k, x \rangle$, $k = 1, 2$; $i, x \in \mathbb{N}$, and

$$\mathbb{N}_0 = \mathbb{N} \setminus \left[\left(\bigcup_{i \in \mathbb{N}} \mathrm{Ran}(\varphi_{1,i}) \cup \left(\bigcup_{i \in \mathbb{N}} \mathrm{Ran}(\varphi_{2,i}) \right) \right] .$$

Definition 5. Let $\{A_k\}_{k \in \mathbb{N}}$ be a sequence of disjoint subsets of $\mathbb{N}_0$. We define the sequence $\{[A_k]\}_{k \in \mathbb{N}}$ of disjoint sets of natural numbers by the following rules:

(a) If $p \in A_i$, then $p \in [A_i]$;

(b) If $1 \leqq l \leqq 2$, $i \in \mathbb{N}$, $p \in [A_k]$ and $\theta_{l,i}(k) = n$, then $\varphi_{l,i}(p) \in [A_n]$.

Lemma 6. *If $\{A_k\}_{k \in \mathbb{N}}$ is a recursive (r.e.) sequence of disjoint subsets of $\mathbb{N}_0$, then $\{[A_k]\}_{k \in \mathbb{N}}$ is a recursive (r.e.) sequence of disjoint sets.*

Lemma 7. *For every natural number x, either $x \in \mathbb{N}_0$ or there exists an effective way to find a function φ which is a composition of the functions $\varphi_{1,0}, \varphi_{2,0}, \varphi_{1,1}, \varphi_{2,1}, \ldots$, and a $y \in \mathbb{N}_0$ such that $\varphi(y) = x$.*

Proof. Using induction on $|x|$, where $|x| = 0$ if $x \in \mathbb{N}_0$ and $|\langle k, i, y \rangle| = |y| + 1$, one can easily verify that Lemma 7 is true.

Lemma 8. *Let $\{A_k\}_{k \in \mathbb{N}}$ be a sequence of disjoint subsets of $\mathbb{N}_0$. Then*

(a) *For any function φ, which is a composition of the functions $\varphi_{1,0}, \varphi_{2,0}, \varphi_{1,1}, \varphi_{2,1}, \ldots$, and for any natural number i there exists such a k that $\varphi([A_i]) \subseteq [A_k]$.*

(b) *For any function φ, which is a composition of the functions $\varphi_{1,0}, \varphi_{2,0}, \varphi_{1,1}, \varphi_{2,1}, \ldots$, and for all distinct natural numbers i, j there exists an effective way to verify whether or not $\varphi([A_i]) \subseteq [A_j]$.*

The proof is immediate.

From now on if $\{A_k\}_{k \in \mathbb{N}}$ is a sequence of disjoint subsets of $\mathbb{N}_0$, then we denote by B_k the set $\bigcup_{i \in M_k} [A_i]$. It is obvious that if $\{A_k\}_{k \in \mathbb{N}}$ is an r.e. sequence, then $\{B_k\}_{k \in \mathbb{N}}$ is r.e. too.

Lemma 9. *If $\{A_k\}_{k \in \mathbb{N}}$ is a sequence of disjoint subsets of $\mathbb{N}_0$, then the following equivalences hold for any natural numbers i and x:*

$$x \in B_i \iff \varphi_{1,2i}(x) \in B_{i+1} \,\&\, \varphi_{2,2i}(x) \notin B_{i+1};$$

$$x \in B_{i+1} \iff \varphi_{1,2i+1}(x) \in B_i \,\&\, \varphi_{2,2i+1}(x) \notin B_i.$$

Proof. This lemma immediately follows from Theorem 1, Lemma 8 and the definitions of A_k, B_k, $k \in \mathbb{N}$.

Corollary. *If $\{A_k\}_{k \in \mathbb{N}}$ is a sequence of disjoint subsets of $\mathbb{N}_0$, then the set B_i is* btt*-equivalent to B_j for all natural numbers i and j.*

Lemma 10. *If $\{A_k\}_{k \in \mathbb{N}}$ is a sequence of disjoint subsets of $\mathbb{N}_0$, and $i \neq j$, then for every sequence $(\psi_1, \ldots, \psi_{l_1}), \ldots, (\psi_1, \ldots, \psi_{l_1})$ of finite sequences of* id *or* *compositions of the functions* $\varphi_{1,0}, \varphi_{2,0}, \varphi_{1,1}, \varphi_{2,1}, \ldots$, *there exists an effective way to find* l, *such that for every* $x \in A_l$ *the equivalence*

$$(***) \qquad x \in B_i \iff (\psi_1(x) \in B_j \;\&\; \ldots \;\&\; \psi_{l_1}(x) \in B_j) \vee \ldots \vee (\psi_{l_{k-1}+1}(x) \in B_j \;\&\; \ldots \;\&\; \psi_{l_k}(x) \in B_j)$$

does not hold.

Proof. This lemma follows again immediately from Theorem 1, Lemma 8 and the definitions of A_k, B_k, $k \in \mathbb{N}$.

Let $\mathbb{N}_0 = \mathbb{N}_1 \cup \mathbb{N}_2$, where $\mathbb{N}_1$ and $\mathbb{N}_2$ are infinite disjoint recursive sets and r' is a monotonically increasing function such that $\mathrm{Ran}(r') = \mathbb{N}_1$ and $r(n) = r'[n.(n+1)/2+n]$.

Additionally, let Φ be a partial recursive function (p.r.f.) which is universal for all unary p.r.f. Let $\Phi_e = \lambda x.\Phi(e,x)$ and $\Phi_{e,s}$ be a finite p.r. approximation of Φ_e, i. e.

$$\Phi_{e,s}(x) = \begin{cases} \Phi_e(x), & \text{if } x \in \mathrm{Dom}(\Phi_e) \;\&\; \Phi_e(x) \text{ is computable in less than } s \text{ steps,} \\ \text{undefined}, & \text{otherwise.} \end{cases}$$

Theorem 2. *There exists an r.e.* btt-*degree which contains an infinite antichain of r.e.* p-*degrees.*

Proof. In order to construct a btt-degree containing infinitely many mutually incomparable p-degrees, we shall construct an r.e. sequence $\{A_k\}_{k\in\mathbb{N}}$ of disjoint subsets of $\mathbb{N}_0$ such that if $i \neq j$, then B_i and B_j are p-incomparable. Then it will follow from Corollary to Lemma 9 that the set B_i has the same btt-degree as the set B_j. Therefore, the proof will be completed.

We construct the sets $\{A_k\}_{k\in\mathbb{N}}$ by steps, building a finite approximation $A_{i,s}$ of A_i, $i \in \mathbb{N}$, on step s. (We shall denote the set $\bigcup_{k\in M_i} [A_{k,s}]$ by $B_{i,s}$.)

At step s, if $(s)_0 = \langle e,i,j\rangle$ and $i \neq j$, our aim is to satisfy the condition that the function Φ_e does not p-reduce B_i to B_j, i. e. to find such an $x \in \mathrm{Dom}(\Phi_e)$ that $\mathrm{Seq}(\Phi_e(x))$, $\forall k[k < \mathrm{lh}(\Phi_e(x)) \Rightarrow \mathrm{Seq}((\Phi_e(x))_k)]$, and at least one of the following two conditions is satisfied:

(i) $x \notin B_i \;\&\; \exists k\{k < \mathrm{lh}(\Phi_e(x)) \;\&\; \forall l[l < \mathrm{lh}((\Phi_e(x))_k) \Rightarrow ((\Phi_e(x))_k)_l \in B_j]\}$;

(ii) $x \in B_i \;\&\; \forall k\{k < \mathrm{lh}(\Phi_e(x)) \Rightarrow \exists l[l < \mathrm{lh}((\Phi_e(x))_k) \;\&\; ((\Phi_e(x))_k)_l \notin B_j]\}$.

For this purpose, on step s, if we find such an x, which satisfies (i), then we would like to put it outside B_i, and if we find an x which satisfies (ii), then we would like to put it in B_i.

If at step s x is placed in some set A_k in order to satisfy either (i) or (ii), then we create an $(s)_0$-*requirement* x. In this case, if x satisfies (ii), then we shall need also some elements $y_1, \ldots, y_p$ which do not belong to any set $[A_k]$. So we create a *negative* $(s)_0$-*requirement* $\{y_1, \ldots, y_p\}$. To guarantee that, for any e, such that Φ_e is total and satisfies the conditions $\forall x[\mathrm{Seq}(\Phi_e(x))]$ and $\forall k[k < \mathrm{lh}(\Phi_e(x)) \Rightarrow \mathrm{Seq}((\Phi_e(x))_k)]$, and for every i, j, such that $i \neq j$, there exists an x satisfying either (i) or (ii), we shall use a priority argument, so that the smaller $(s)_0$ will have priority.

If x is an $(s)_0$-requirement and $\{y_1, \ldots, y_p\}$ is a negative $(s)_0$-requirement created at step s, and till step t the condition (ii) which is satisfied at step s is not injured, then we shall say that the $(s)_0$-requirement and the negative $(s)_0$-requirement are *active* at step t.

If an $(s)_0$-requirement x satisfies (i), then we call it *active* at every step $t > s$.

If an $(s)_0$-requirement (a negative $(s)_0$-requirement) created at step s is active at every step $t > s$, then we say that it is *constant*.

Now we can describe the construction of the sequence $\{A_k\}_{k\in\mathbb{N}}$.

Step $s = 0$. Let $\mathbb{N}_2 = \{a_0, a_1, \ldots\}$, where $a_0 < a_1 < \ldots$; we take $A_{i,0} = \{a_i\}$. Thus it is ensured that A_i is nonempty.

Step $s > 0$. If not $\mathrm{Seq}((s)_0)$ or $[\mathrm{Seq}((s)_0)$ and $((s)_0)_1 = ((s)_0)_2]$, then we do nothing, i.e. we take $A_{i,s} = A_{i,s-1}$, $i \in \mathbb{N}$, and do not create any requirements.

If $\mathrm{Seq}((s)_0)$ and $(s)_0 = \langle e, i, j\rangle$, where $i \neq j$, we verify whether an active $(s)_0$-requirement exists. If there exists such a requirement, then we do nothing.

If such a requirement does not exist, then we verify whether there exists an $x \in \mathbb{N}_1$ such that

$$x > r((s)_0), \quad x \in \mathrm{Dom}(\Phi_{e,s}), \quad \mathrm{Seq}(\Phi_e(x)),$$

$$\forall k[k < \mathrm{lh}(\Phi_e(x)) \Rightarrow \mathrm{Seq}((\Phi_e(x))_k)], \quad x \notin \bigcup_{i\in\mathbb{N}} A_{i,s-1},$$

and x does not belong to any active negative requirement, created at a step $t < s$ such that $(t)_0 < (s)_0$. If such an x does not exist, then we do nothing.

Otherwise we denote by x_s the least such x and create the $(s)_0$-requirement x_s. Let

$$\Phi_e(x_s) = \langle\langle z_1, \ldots, z_{l_1}\rangle, \ldots, \langle z_{l_{k-1}+1}, \ldots, z_{l_k}\rangle\rangle, \quad \psi_p(y_p) = z_p,$$

where either ψ_p is a composition of the functions $\varphi_{1,0}$, $\varphi_{2,0}$, $\varphi_{1,1}$, $\varphi_{2,1}$, $\ldots$, or $\psi_k = \mathrm{id}$, $1 \leqq p \leqq l_k$, and $y_1, \ldots, y_{l_k} \in \mathbb{N}_0$. We verify, whether there exist natural numbers $z_{i_1}, \ldots, z_{i_k}$ such that $z_{i_p} \notin B_{j,s-1}$, $l_{p-1}+1 \leqq i_p \leqq l_p$ and $x_s \neq y_{i_p}$, $p = 1$, $\ldots$, k. If yes, then

$$A_{p_i,s} = A_{p_i,s-1} \cup \{x_s\}, \quad A_{l,s} = A_{l,s-1} \quad \text{for } l \neq p_i, \ l \in \mathbb{N},$$

and if $\{y_{i_1}, \ldots, y_{i_k}\} \setminus \left(\bigcup_{l\in\mathbb{N}} A_{l,s-1} \cup \{x_s\}\right)$ is nonempty, we create a negative $(s)_0$-requirement

$$\{y_{i_1}, \ldots, y_{i_k}\} \setminus \left(\bigcup_{l\in\mathbb{N}} A_{l,s-1} \cup \{x_s\}\right).$$

Otherwise we consider all those p, $1 \leqq p \leqq k$, such that there does not exist an i_p such that $z_{i_p} \notin B_{j,s-1}$, $l_{p-1} + 1 \leqq i_p \leqq l_p$, and $x_s \neq y_{i_p}$. Let us assume in this case that all these p are 1, $\ldots$, q and i_{q+1}, $\ldots$, i_k are such that $z_{i_p} \notin B_{j,s-1}$, $l_{p-1} + 1 \leqq i_p \leqq l_p$, $y_{i_p} \neq x_s$, $p = q+1$, $\ldots$, k. For any p, $1 \leqq p \leqq q$, we consider all those i such that $l_{p-1} + 1 \leqq i \leqq l_p$ and $y_i = x_s$. We assume that for any p, $1 \leqq p \leqq q$, all those i, such that $l_{p-1} + 1 \leqq i \leqq l_p$ and $y_i = x_s$, are $l_{p-1}+1, \ldots, l_p$.

According to Lemma 10 there exists an l such that if $p \in A_l$, then the equivalence $(***)$ does not hold if we replace k by q. We define

$$A_{l,s} = A_{l,s-1} \cup \{x_s\} \text{ and } A_{p,s} = A_{p,s-1} \quad \text{for } p \neq l, \ p \in \mathbf{N},$$

and we create a negative $(s)_0$-requirement

$$\{y_{i_{q+1}}, \ldots, y_{i_k}\} \setminus \Big(\bigcup_{l \in \mathbf{N}} A_{l,s-1} \cup \{x_s\} \Big)$$

if $\{y_{i_{q+1}}, \ldots, y_{i_k}\} \setminus \Big(\bigcup_{l \in \mathbf{N}} A_{l,s-1} \cup \{x_s\} \Big)$ is nonempty.

Finally, we take $A_i = \bigcup_{s \in \mathbf{N}} A_{i,s}$.

Obviously, this construction is effective, hence the sequence $\{A_k\}_{k \in \mathbf{N}}$ is r.e. Moreover, $\{A_k\}_{k \in \mathbf{N}}$ is a sequence of disjoint subsets of $\mathbf{N}_0$, since one element may be placed in only one A_k.

In order to show that this construction works, we need some lemmas. Let $A = \bigcup_{i \in \mathbf{N}} A_i$.

Lemma 11. *The set $\mathbf{N}_1 \setminus A$ is infinite.*

Proof. Let $(\mathbf{N}_1)_n = \{x \mid x \in \mathbf{N}_1 \,\&\, x < r'(n)\}$. We prove that the set $(\mathbf{N}_1)_{r(n)} \cap (\mathbf{N}_1 \setminus A)$ contains at least n elements or, equivalently, $|(\mathbf{N}_1)_{r(n)} \cap A| \leqq n.(n+1)/2$.

Indeed, for every $\langle e,i,j \rangle$, $i \neq j$, we have no more than $\langle e,i,j \rangle + 1$ $\langle e,i,j \rangle$-requirements and each of them is greater than $r(\langle e,i,j \rangle)$ and belongs to some $A_k \subseteq A$. Therefore, in $(\mathbf{N}_1)_{r(n)} \cap A$ there are only m-requirements for $m < n$, i. e. in $(\mathbf{N}_1)_{r(n)} \cap A$ there are no more than $1 + 2 + \cdots + n = n.(n+1)/2$ elements.

Lemma 11 is proved.

Lemma 12. *The set $\mathbf{N}_1 \setminus A$ is immune.*

Proof. Let us assume that there exists a set $C \subseteq \mathbf{N}_1 \setminus A$ which is infinite and r.e., and $x_0 \in \mathbf{N}_2$. Obviously,

$$f(x) = \begin{cases} \langle\langle x_0 \rangle\rangle, & \text{if } x \in C, \\ \text{undefined}, & \text{otherwise} \end{cases}$$

is a p.r.f. Let e be a natural number such that $f = \Phi_e$, and let $x \in \mathrm{Dom}(f)$ such that $x > r(\langle e,0,1 \rangle)$ and s_0 is the least s which satisfies the equality $\Phi_{e,s}(x) = f(x)$. Then x must be an $\langle e,0,1 \rangle$-requirement created at some step $s > s_0$ such that $(s)_0 = \langle e,0,1 \rangle$, i. e. $C \cap A$ is nonempty. This contradicts the assumption. Therefore, $\mathbf{N}_1 \setminus A$ is immune.

Lemma 13. *For any natural number e such that $\mathbf{N}_1 \subseteq \mathrm{Dom}(\Phi_e)$ and*

$$\forall x \{x \in \mathbf{N}_1 \Rightarrow \mathrm{Seq}(\Phi_e(x)) \,\&\, \forall k [k < \mathrm{lh}(\Phi_e(x)) \Rightarrow \mathrm{Seq}((\Phi_e(x))_k)]\},$$

and for all distinct i, j, there exists a constant $\langle e,i,j \rangle$-requirement.

Proof. Assume that there is no constant $\langle e,i,j \rangle$-requirement, where $i \neq j$, $\mathbf{N}_1 \subseteq \mathrm{Dom}(\Phi_e)$ and

$$\forall x \{x \in \mathbf{N}_1 \Rightarrow \mathrm{Seq}(\Phi_e(x)) \,\&\, \forall k [k < \mathrm{lh}(\Phi_e(x)) \Rightarrow \mathrm{Seq}((\Phi_e(x))_k)]\}.$$

We find an s_0 such that if $s \geqq s_0$ and $\langle e_1, i_1, j_1\rangle < \langle e, i, j\rangle$, then every $\langle e_1, i_1, j_1\rangle$-requirement is already created. Moreover, let $x \in \mathbf{N}_1 \setminus A$, $x > r(\langle e, i, j\rangle)$ and s be such that $s \geqq s_0$, $\Phi_{e,s}(x) = \Phi_e(x)$ and $(s)_0 = \langle e, i, j\rangle$. Then on step s a constant $\langle e, i, j\rangle$-requirement x is created.

Lemma 13 is proved.

Now we shall prove Theorem 2. Let us assume that $B_i \leq_{\mathrm{p}} B_j$ and $i \neq j$. Therefore, there exists a total recursive function f such that

$$\forall x\{\mathrm{Seq}(f(x))\,\&\,\forall k[k < \mathrm{lh}(f(x)) \Rightarrow \mathrm{Seq}((f(x))_k)]\},$$

and

$$\forall x\{x \in B_i \iff \exists k[k < \mathrm{lh}(f(x))\,\&\,\forall l(l < \mathrm{lh}((f(x))_k) \Rightarrow ((f(x))_k)_l \in B_j)]\}.$$

Let e be an index such that $\Phi_e = f$. It follows from Lemma 13 that there exists a constant $\langle e, i, j\rangle$-requirement x_s created at step s. Then $x_s \in \mathbf{N}_1$,

$$f(x_s) = \langle\langle z_1, \ldots, z_{l_1}\rangle, \ldots, \langle z_{l_{k-1}+1}, \ldots, z_{l_k}\rangle\rangle, \quad \psi_p(y_p) = z_p,$$

where ψ_p is either a composition of the functions $\varphi_{1,0}$, $\varphi_{2,0}$, $\varphi_{1,1}$, $\varphi_{2,1}$, $\ldots$, or $\psi_p = \mathrm{id}$, $p = 1, \ldots, l_k$, and $y_1, \ldots, y_{l_k} \in \mathbf{N}_0$. We assume that there exist natural numbers $z_{i_1}, \ldots, z_{i_k}$ such that $z_{i_p} \notin B_{j,s-1}$, $l_{p-1} + 1 \leqq i_p \leqq l_p$ and $x_s \neq y_{i_p}$, $p = 1, \ldots, k$. This contradicts the fact that the function f p-reduces B_i to B_j. Therefore, there exists a p, $1 \leqq p \leqq k$, such that there does not exist i_p such that $z_{i_p} \notin B_{j,s-1}$, $l_{p-1} + 1 \leqq i_p \leqq l_p$ and $x_s \neq y_{i_p}$. Let all those p be $1, \ldots, q$ and $z_{i_p} \notin B_{j,s-1}$, $l_{p-1} + 1 \leqq i_p \leqq l_p$, and $x_s \neq y_{i_p}$, $p = q + 1, \ldots, k$. For the sake of simplicity we shall assume that $y_1 = \cdots = y_{l_q} = x_s$. Then it is easy to check that x_s does not satisfy the condition (p) if we replace x with x_s, A with B_i, and B with B_j, which contradicts the fact that f p-reduces B_i to B_j.

Theorem 2 has been proved.

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