NIKOLA OBRESHKOFF (1896–1963)

ENCOMIUM*

BOYAN PENKOV

It is a honouring and pleasant duty to express my thanks to the organizers of this memorial meeting for having invited me to hold an introductory encomium on the occasion of the centenary of Nikola Obreshkoff, who was and continues to be a significant phenomenon in Bulgarian mathematics.

Please do not interprete my first words as an attempt to justify myself before-hand when confessing that I was confronted with obstacles, most of which pleasing but difficult to overcome. The first obstacle has been formulated by Goethe in the words of Faust:

Ach, die Erscheinung war so riesengross,

dass ich mich recht als Zwerg empfinden sollte.

The Bulgarian translation from 1905 of the seventeen years older colleague and friend Alexander Balabanov (another great phenomenon at the then Bulgarian horizon) of Goethes lines is as follows:

Видението бе до небеса́,

а аз пред него бях играчка само.

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* Invited address at the memorial meeting on the occasion of the 100th anniversary of N. Obreshkoff held at the Bulgarian Academy of Sciences on April 18, 1996. The original talk was held in Bulgarian. This is an English translation of the author.

Goethe has grasped fairly deep how difficult it is to keep the distance to somebody excelling you and yet to try to get knowing him better. So much about the first obstacle.

The second obstacle is related to time — more precisely, the time that has passed. And this kind of time is always long enough but also insufficient. Some things languish in oblivion, some others have not yet settled down to be declared history. It is now 33 years that Obreshkoff is not among us. He suddenly passed away in the late summer of 1963 and just a month later was followed by Lyubomir Tchakalov. Some colleagues called it the "black autumn" of Bulgarian mathematics. 33 years equals the age of Jesus and the span of a generation. The number of colleagues having seen Obreshkoff live can be counted today on the fingers of your hands.

And last the third obstacle. We live in a country deprived of memory. How many are our citizens who can cite the birthdates of their grandparents, how many family, municipal or institutional archives are being preserved? To commemorate people like Obreshkoff would be quite easier if there were in this country professional historians of mathematics, if such a subject was part of the curricula of the now so many math departments and was not only taught but was also an object of research. The fragmentary and pale efforts in this direction cannot fill the institutional gap. Lonely enthusiasts have repeatedly tried to change the situation (it suffices to recall the name of the late Boyan Petkantchin) but their voices faded away in the wilderness.

Obreshkoff's creative activity spans over a 40 year period, from the beginning of the twenties till the beginning of the sixties. The life of a genuine mathematician — Obreshkoff was such one *par excellence* — consists of his research results. They have been announced in about 250 publications. The average number of papers published by Obreshkoff yearly is 6 or 7, the minimum of 2 papers falls at WW2 years 1944 and 1945 and the maxima — in 1938 (10 works) and 1949 (12 works). 74 of these papers are by now collected in the first two volumes of Obreshkoff's *Collected Works* that started to appear in 1977 and stopped without any arguments in 1981, when the third volume, ready to be printed, did not reach the Publishing house of the Academy of Sciences. There is one more mystery around this edition. The first volume, out of print for a long time, was republished by the renowned editing house *Birkhäuser* in Basel together with an announcement for several further volumes. The Bulgarian mathematical community has not seen this republished volume.

Before starting the risky adventure to cast a bird's eye view over the mathematical problems that Obreshkoff has dealt with — they will be discussed in detail tomorrow at a special session at the mathematical department — let me remind you the main points of his CV.

Nikola Dimitrov Obreshkoff was born in the town of Varna on April 18th, 1896 as one of the last children in a large and bright family. The father, born in 1858, was a military officer, achieving later the rank of a colonel. The mother of ten children Kitza Obreshkova — a music lover and fluent in French, was the moral and intellectual force of the family. With the beginning of this century the family moved to the capital Sofia, where Nikola graduated in 1915 from the Second

Sofia Boys High-school. Three years earlier the 16 years old high-school student published in Vol. 8 of the *Journal of the Bulgarian Physico-mathematical Society* a paper entitled *Expressing functions of half angles through functions of whole angles*. In the fall of 1915 Nikola was admitted student in mathematics and physics at the Physico-mathematical Department of Sofia University. The First World War interrupted his studies temporarily and he served as private and later as officer in a field engineering unit. Immediately after graduating in 1920 he was appointed assistant at the Chair of Calculus. In this position he was conducting practical works with the students not only in calculus but in other subjects too, that was something common for the time, but not for nowadays. Even 25 years later most of the assistants were multipresent and worked on many different math courses, at least on two. At that time the Chair was not an organisational unit but an area for which an ordinarius (full-time professor) was responsible. Let me leave it to you to decide what kind of progress is the todays almost impossibility to ask an assistant from the Chair of Algebra to conduct practical work in calculus for, say, freshmen. Reflecting on the works of Obreshkoff, a difficult question arises: was he an algebraist or an analyst or, say, a probabilist. He was all of this together.

After two years of assistantship Obreshkoff got his 'Habilitation' in 1922 as an 'ordinary docent' (= assistant professor) with his papers on distribution of zeros of polynomials, his first love to which he remained faithful to his last gasp. One of the reviewers was Kyrill Popoff. His review reads as follows:

"Delighted by the results [of Obreshkoff] I communicated them to Prof. Dr. Issai Schur, ordinarius for higher algebra at Berlin University. Here are his impressions and his opinion on the value of the paper considered [the Habilitation schrift] expressed in a letter, which I am citing here with his kind permission:

<div align="right">Berlin, den 13 September 1921</div>

Sehr gechrter Herr Kollege!
Die Arbeit des Herrn N. Obreschkoff "Über die Verteilung der Wurzeln der algebraischen Gleichungen", die Sie die Freundlichkeit hatten, mir zu überbringen, hat mich sehr interessiert. Die von Herrn Obreschkoff angegebene Erweiterung des Budan-Fourierschen Theorems auf das Komplexe Gebiet ist von bemerkenswerter Eleganz und Einfachkeit. Bedarf die Beweisführung auch noch einer erheblichen Kürzung, so zeugt die Arbeit doch von dem Scharfsinn des Verfassers und sein Resultat stellt einen wertvollen Beitrag zur Theorie der algebraischen Gleichungen dar.

<div align="right">Mit hochachtungsvollen Grüßen Ihr sehr ergebener</div>
<div align="right">Prof. Dr. I. Schur."</div>

And Popoff continues:

"The Habilitation schrift of Mr. Obreschkoff is a valuable contribution to the field of Higher Algebra, revealing his big talent and assuring him a leading position among the young mathematicians. It shows original thought, gift to

see by himself the fundamental issues and to achieve the solution by his own efforts. All this is demonstrated also by his paper on series, though not solving problems of the same importance as the above mentioned, it shows a formed mathematical insight and an outright individuality.

I do recommend warmly Mr. Obreshkoff for the position of assistant professor at the Chair of Higher algebra.

> Dr. Kyrill Popoff
> Associate Professor of
> Differential and Integral Calculus."

In 1925 Obreshkoff was promoted an associate professor and in 1928 — a full-time professor and Head of the Algebra Chair. He remained at this position for 35 years.

As a young lecturer he had different courses. According to the then terminology, some 'basic' ones: Higher algebra (in two parts), Infinite series, Theory of probability, and some 'temporary': Spherical and practical astronomy, Plane analytic geometry, Differential geometry.

Obreshkoff has never been abroad for a long time as a postgraduate. His two Ph.D. degrees — from Palermo and Paris, he got in 1932 and 1933, being yet a full-time professor and author of more than thirty publications.

In order to accomplish this dry recording of facts, allow me, please, a digression. I cited above the report of Popoff, the other reviewer was Emanuel Ivanoff, the then Head of the Algebra Chair. Both reports are printed in vol. 19 (1922) of the *Annuaire de l'Université de Sofia* and thus were immediately made open to the public, together with the inaugural lecture of the newly elected professor, entitled: *Character and Problems of Algebra*. Grace to such a publicity, it is not a secret to us today who did recommend and with what arguments Obreshkoff's promotion. The responsibility which Ivanoff and Popoff assumed in 1921 we count today to their merits. In later times, especially after the forties, unfortunately things became anonimous, the reports of the reviewers being available only to a restricted circle of scholars and the original documents sinking into the archives (if not destroyed) and not made known to the general public. Try nowadays to discover who proposed whom for a certain academic position! Let us hope that the Bulgarian Academic society will realize the necessity of such a publicity, lost half a century ago.

In January 1945 Obreshkoff was elected directly as an Ordinary member of the Bulgarian Academy of Sciences and Arts, as it was called at that time. The usual path was through becoming first Corresponding member, but Obreshkoff was elected directly Ordinary member. Earlier members were Ivan Tzenoff (elected in 1929) and Lyubomir Tchakaloff (elected in 1930). I do not count here the mathematicians Vassil Vassilieff, Ivan Gyuzeleff, Emanuel Ivanoff and Georgi Kirkoff, who were members of the Bulgarian Literary Society, the ancestor of the Academy.

After the sovietisation of the Academy through the Acts of 1947 and 1949 and the foundation of some research institutes in the framework of the Academy Obreshkoff was appointed in 1950 as the first director of the recently created Mathematical Institute. The death overtook him after 13 years in this position at a

crucial moment of the institute's development. As Marshall Stone has formulated it, mathematics has turned to be not only a vocation, but a profession. Some times earlier cybernetics (as the computer sciences were called) was declared sane and removed from the 'index scientiarum prohibitorum'. Mathematical methods in the social sciences and in the humanities were accepted. The application of mathematics in industry, even the intention to go in this direction, became fashionable. Obreshkoff was not allowed to participate in this 'taw' development. I have remembered his directorship (up to 1955) by two main characteristics — at first place his absolute intolerance of low quality research, and second, his highly developed sense of responsibility concerning public affairs. And all this accompanied by an inborn allergy towards bureaucracy. It was the essence, not the form that did matter for him. He often repeated that this country is small and poor and we have to economize in everything and everywhere, and that in the field of research the then loudly proclaimed law that 'quantity develops into (of course better) quality' was false.

Dear colleagues and esteemed audience! To discuss here, even minimizing the details, the rich and diverse works of Obreshkoff is not possible and beyond my scope.

Let me try to present a very brief and general sketch. Kyrille Popoff used to say that one is working during all his life on his dissertation, literally on his 'thesis'. To some extent this applies to Obreshkoff, too. He impressed the mathematical community with his very first paper on the distribution of zeros of polynomials. His beautiful generalization of the theorem of Budan and Fourier was achieved by generalizing a lemma of Johann von Segner from the midst of the 18th century. In Segner's original proposition the factor is linear, but Obreshkoff used a quadratic one, thus generalizing Descartes' rule of signs to complex valued zeros. During the twenties and thirties of this century the distribution of values of polynomials was a busy research area and Obreshkoff was one of the prominent dramatis personae, along with Dieudonné, Faber, Féjér, Fujiwara, Kakeya, Marden, Montel, Polya, Schoenberg, Schur, Szökefalvi-Nagy, Szegö, Turan, Walsh et al. As yet mentioned, Obreshkoff did not abandon these problems until his last days. Only few months before his unexpected death two monographs were published: *Zeros of Polynomials* (in Bulgarian, Sofia) and *Verteilung und Berechnung der Nullstellen realer Polynome* (in German, Berlin). These books are the result of 40 years of active research in this field. Earlier there were published only two monographs in this area: Nr 93 of *Mémorial des sciences mathématiques* by Dieudonné (1938) and *Geometry of polynomials* by Marden (1949). The first volume of Obreshkoff's Collected Works contains 45 papers upon zeros.

In his inaugural lecture by "the basic problem of algebra" Obreshkoff meant the solution of algebraic equations. Nowadays the term "algebra" has a quite different meaning. The distribution of zeros of polynomials belongs therefore to the domain of analysis. Dieudonné's review of 1938 is called: *Théorie analytique des polynomes d'une variable*.

Obreshkoff contributed also to the distribution of zeros of entire functions, to particular meromorphic functions which are limits of special polynomials or rational functions. These results interfere with his interest in functional series and lead him

to his second great love that turned out to be very fruitful: the summation of divergent series. Unfortunately, he did not succeed to present this part of his life work in a bookform. But the bifurcations from this theory are very interesting. In his famous paper on quadrature formulae, published in the *Proceedings of the Prussian Academy of Sciences* in 1940, the approach is based on a summation formula.

During the second half of the forties Obreshkoff achieved a brilliant result in diophantine approximations and gave the answer to a problem posed by Borel as early as 1903. Obreshkoff proved that the unknown 'Borel constant' is equal to 1.

Last but not least we should not forget that Obreshkoff has interesting contributions to the probability theory (series and polynomials of Charlier connected with the Poisson distribution). They are published in the series *Actualités Scientifiques et industrielles* in 1938.

Obreshkoff must be mentioned also as the author of many and influential textbooks. In a short period the young professor published as № 93, 110 and 153 (resp. in 1930, 1932 and 1935) of the famous Bulgarian *University library* series two volumes of Higher algebra and a Collection of problems in the same field. Within 25 years the Higher algebra underwent more than five editions. But comparing the first edition (1930) with the last one (1955) you will notice the richness of the first. It contains: fundamental properties of polynomials, determinants, basic properties of algebraic equations, algebraic solution of equations, theory of numbers, theory of groups and its applications to algebraic solution, theory of Galois and finally Abel's theorem. The later editions are somehow simplified, they contain linear algebra, but some deeper topics are omitted. The second volume of this algebra textbook is in fact the first textbook on probability and statistic written by a professor of Sofia University, parallel to Oskar Anderson's (the then director of the Economical Research Institute at the University) *Einführung in die mathematische Statistik* from 1935. During the fifties the two initial volumes of the Higher algebra (which at least to me are still charming and challenging) were split among others into textbooks on probability and theory of numbers.

One can meet the name of Obreshkoff also as author of some highschool textbooks and two popular booklets (one on Euler, with co-author Yordan Duitchev, and the another under the title *What is differentiating?* with co-author Dimiter Skordev). These nice texts remind me of Herbert Robbins' joke about his co-authorship with Courant on *What is mathematics*. The version was that Courant wrote the text but put on the front page the prestigious name of young Robbins, as Hilbert did with Courant in *Methoden der mathematischen Physik*.

Tomorrow and after tomorrow during the specialised session many of you will have the possibility of following the chalk on the blackboard (the good old way to communicate mathematical ideas) to learn more on Obreshkoff's works on integral transforms and many other things. Therefore allow me to skip them here.

And now, after these words, you will be able to hear some reminiscences on the human being Obreshkoff and I shall myself not elaborate on his image that was in a moving manner unsophisticated. He had no hidden or surprising facets, but was both direct and kind. Not alien to public problems, nevertheless he was absorbed

by his internal mathematical world. I do not remember him in a bad mood, even after his physical pains became more frequent in the late fifties. He was not a lecturer for beginners but an excellent one for advanced students. This feature he had in common with Kolmogorov — they shared a creative manner of speaking and their words could be decoded only by the initiated. One more resemblance between them was that scarcely you had shared a problem you could realize they had gone through it and as Obreshkoff used to say: 'I have been thinking about this'. Indeed there were many things he had thought about.

The mathematical community of this country still owes much to Obreshkoff. We have to accomplish the edition of his complete works and we must compilate his scientific biography.

It is fine that Sofia has now an Obreshkov street, but his hospitable home at Tzar Samuel street deserves since a long time a memorial plate.

The best what future generations of Bulgarian mathematicians can do to honour the memory of Obreshkoff is to be exacting and persevering like him.

23 Oborishte str.
BG 1504 Sofia, Bulgaria
E-mail: bip@math.acad.bg